The University of Alabama at Birmingham (UAB)
Department of Physics

PH 461/561 – Classical Mechanics I – Fall 2005

<u>Assignment # 6</u> Due: **Thursday, October 6**
(Turn in for credit!)

<u>Activities based on previous lectures:</u>

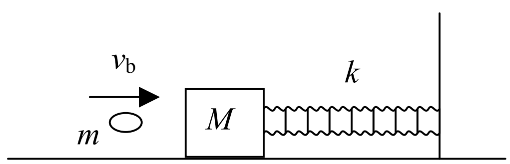
1. A particle of mass describes harmonic motion with an amplitude A. Knowing that the spring constant is k, find the following:

    a. The total energy of the particle.
    b. A relationship between the potential energy and the total energy as a function of the displacement from the position of equilibrium x.
    c. The ratio of the kinetic energy to the total energy as a function of the displacement from the position of equilibrium x.

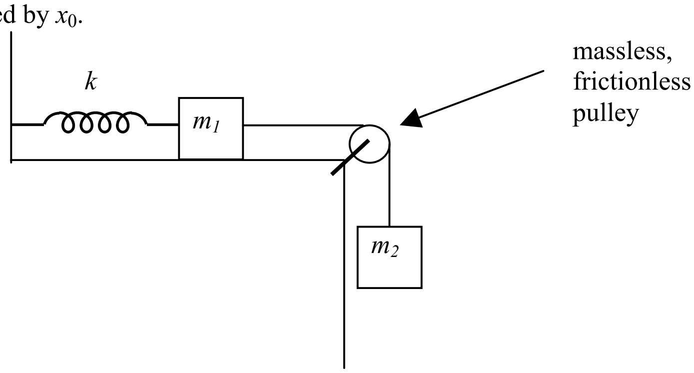
2. You are told that, at the known positions x_1 and x_2 an oscillating mass m has speeds v_1 and v_2. What are the amplitude and the angular frequency of the oscillations?

3. A block of mass M, at rest on a horizontal frictionless table, is attached to a rigid support by a spring of constant k. A bullet of mass m moving with a speed v_b strikes the block as shown in the figure below. The bullet is embedded in the block.

    a. Determine the speed of the block immediately after the collision.
    b. The amplitude of the resulting simple harmonic motion.
    c. The period of the resulting simple harmonic motion.



4. Blocks of masses m_1 and m_2 are connected as shown in the figure below. The surface on which m_1 slides is frictionless, as is the massless pulley. The spring and cord which connects the blocks have negligible mass. Determine the positions of the masses if the system was released from rest when the spring was stretched by x_0.



5. The potential energy of two atoms in a molecule can sometimes be approximated by the Morse function,

$$U(r) = A\left[\left(e^{(R-r)/S} - 1\right)^2 - 1\right]$$

where r is the distance between the two atoms and A, R, and S are positive constants with $S << R$.

   a. Sketch this function for $0 < r < \infty$.
   b. Find the equilibrium separation r_0, at which $U(r)$ is minimum.
   c. Now write $r = r_0 + x$ so that x is the displacement from equilibrium, and show that, for small displacements, U has the approximate form $U(x) = \frac{1}{2}kx^2 + const$ (This means that Hooke's law applies.)
   d. What is the force constant k?
   e. What is the period of small oscillations?

6. Consider a simple harmonic oscillator with period τ. Let $\langle f \rangle$ denote the average value of any variable $f(t)$, averaged over one complete cycle:

$$\langle f \rangle = \frac{1}{\tau}\int_0^\tau f(t)dt$$

Because the instantaneous position $x(t)$ and velocity $v(t)$ of a harmonic oscillator change with time, its potential energy V and kinetic energy T are also functions of time.

   a. Find expressions for the time dependence of V and T.
   b. Determine the average value $\langle V \rangle$ of V over one oscillation cycle.
   c. Determine the average value $\langle T \rangle$ of T over one oscillation cycle.
   d. Prove that $\langle T \rangle = \langle V \rangle = \frac{1}{2}E$ where E is the total energy of the oscillator.
   e. Show also that the average total energy of the oscillator (i.e., $\langle E \rangle = \langle V \rangle + \langle T \rangle$) is equal to the instantaneous total energy.

7. A massless spring has unstretched length l_0 and a force constant k. One end is now attached to the ceiling and a mass m is hung from the other. The equilibrium length of the spring is now l_1.

   a. Write down the condition that determines l_1.
   b. Suppose now that the spring is stretched a further distance x beyond its new equilibrium length. Show that the net force (spring plus gravity) on the mass is $F = -kx$. That is, the net force obeys Hooke's law, when x is the distance from the equilibrium position (This is a very useful result, which lets us treat a mass on a vertical spring just as if it were horizontal).
   c. Prove the same result by showing that the net potential energy (spring plus gravity) has the form $V(x) = \frac{1}{2}kx^2 + const$.

8. The potential energy for the one-dimensional motion of a mass m at a distance r from the origin is

$$U(r) = U_0 A\left(\frac{r}{R} + \lambda^2 \frac{R}{r}\right)$$

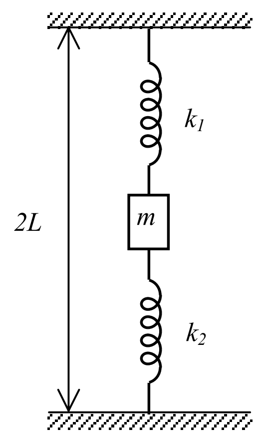
for $0 < r < \infty$, with U_0, R, λ all positive constants.

   a. Find the equilibrium position r_0.
   b. Let x be the distance from equilibrium and show that, for small x, the potential energy has the form $U = \frac{1}{2}kx^2 + const$.
   c. What is the angular frequency of small oscillations?

9. Fowles and Cassiday 7th Edition, Problem 3.7.

10. Find expressions for the frequency of oscillation f for the series and parallel arrangements of problem 9 above (Note: $f = \omega_0/2\pi$).

12. Calculate the frequency of free oscillations of the system below assuming that the body of mass m can only move in the vertical direction and it is subject to a constant force of gravity. Neglect air resistance and assume the springs are massless.



13. An object of mass M is connected to the end of a spring with spring constant k and mass m. The object oscillates in the vertical direction with and there is negligible air resistance. Find the period of oscillations of this system as a function of M, m, and k.

14. Fowles and Cassiday 7th Edition, Problem 3.8.